

FILTRATION OF NON-NEWTONIAN LIQUIDS

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An analysis of theoretical and experimental data relating to non-Newtonian filtration is presented. A filtration equation is proposed for suspensions.

The filtration of non-Newtonian liquids in order to separate the solid particles of a suspension from a polymer solution or melt is now beginning to acquire industrial significance.

There are two quite different methods of separation by filtration. The first relates to the separation of dilute and colloidal suspensions and is based on the settling of the solid particles on the well-developed surface of the channels inside the filtering layer. The second is appropriate for separating suspensions with a high concentration of the solid phase and is based on the sifting action of the filtering barrier. During filtration the solid particles form a layer of deposit on the surface of the filter fabric. This method applies to non-Newtonian liquids and is the one which will be considered in this paper. Up to the present time only a few papers have been devoted to this subject [1-8].

Theoretically, filtration may be represented in the form of the motion of liquid through a porous layer of variable thickness. However, such a representation does not correspond to the true course of the process even for simple non-Newtonian liquids. The layer of filtering deposit is usually compressible and consists of irregularly shaped grains with a broad size distribution. In spite of this fact the simplified solutions of the equations of motion in the filtering layer are still very useful. From these we may derive an equation for a model of the process, containing empirical parameters with a specific physical meaning. In the ensuing discussions we shall assume the following conditions regarding the filtration process, the properties of the liquid, and the mechanism of its motion: The rate of sedimentation of the solid particles in the suspension may be neglected; the filtrate is an incompressible Stokes liquid; the motion of the liquid through the deposit has a laminar character; in view of the extremely low velocity of the liquid we may use the capillary model of its motion based on the concept of the hydraulic diameter.

The equation of one-dimensional filtration of a non-Newtonian liquid through a compressible residue (generalized Darcy equation) may be expressed in the form [5]

$$q_x = (u_x - e_x r_x - u_{px}) \frac{K}{\mu_{ef}} \left(\frac{dp}{dx} \right). \quad (1)$$

In Eq. (1) the direction of motion of the liquid is opposite to the direction of the coordinate axis (Fig. 1). The quantity q_x depends on the distance from the filtration barrier, since the volume of the filtrate increases on account of the liquid displaced from the deposit by reason of its compressibility. It should be emphasized that the effect of the motion of the solid grains in the deposit is only appreciable in suspensions with a high concentration of the solid phase.

The question as to the existence of boundary effects during the filtration of a polymer solution represented in Eq. (1) by the apparent slip velocity u_{px} remains contentious. Some authors [2], however, insist on the existence of slip during motion through the granular layer as a result of the adsorption of the polymer on the surface of the solid particles.

The effective viscosity may be determined on the basis of any model of the liquid and type of motion through the packing.

Since in the filtration of non-Newtonian liquids one most frequently encounters suspensions with a low

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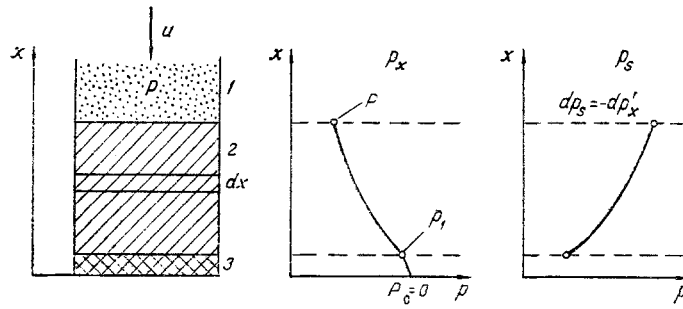


Fig. 1. Schematic representation of the process: 1) suspension; 2) deposit; 3) filtration barrier.

content of the solid phase, in the majority of cases the filtration velocity u_x along the direction of flow will be approximately constant and equal to the velocity at the surface of the filtration barrier u_1 . Neglecting possible boundary effects, which are only of formal significance in Eq. (1) on account of later simplifications, we obtain

$$q_x = u_1 = \frac{dV}{dt} = \frac{K}{\mu_{ef}} \left(\frac{dp}{dt} \right). \quad (2)$$

Equivalent to this formulation for liquids obeying a rheological power law

$$\tau = k\gamma^n \quad (3)$$

is the simple Christopher — Middleman equation [9]

$$u = \left[\frac{k}{\psi} \cdot \frac{dp}{dx} \right]^{1/n}, \quad (4)$$

where

$$\psi = \frac{k}{12} \left(9 + \frac{3}{n} \right) (150K\epsilon)^{\frac{1-n}{2}}.$$

Using the definition of the permeability coefficient of the deposit and the relationship between the thickness of the layer of deposit, its mass, and the volume of the filtrate, Eq. (4) may be brought to the form

$$u_1 = \left(\frac{1}{k\chi_x} \cdot \frac{dp}{dw_x} \right)^{1/n}. \quad (5)$$

In Eq. (5) χ_x represents the modified specific resistance of the deposit determined by the equation

$$\chi_x = \alpha_x^{\frac{n+1}{2}} \left(\frac{3n+1}{4n} \right)^n \left[\frac{2\rho_s(1-\epsilon_x)}{T\epsilon_x} \right]^{\frac{n-1}{2}}. \quad (6)$$

For a Newtonian liquid, Eq. (6) reduces to the well-known equation

$$n = 1, \quad \chi_x = \alpha_x = \frac{2TS^2(1-\epsilon_x)}{\rho_s\epsilon_x^3}. \quad (7)$$

We introduce the differential of the pressure compressing the deposit (Fig. 1) into Eq. (7), and assuming that the specific resistance χ_x is a function of the compressing pressure, integrate it over the whole thickness of the deposited layer ($w_x = 0$; $w_x = w$):

$$\int_0^{P-P_1} \frac{dp_s}{\chi_x} = k(u_1)^n w. \quad (8)$$

For non-Newtonian filtration the average modified specific resistance χ_R is calculated from the equation

$$\chi_R = \frac{P - P_1}{\int_0^{P-P_1} \frac{dP}{d\chi_x}}. \quad (9)$$

Taking the pressure of the liquid on the outer side of the filtration barrier as zero ($P = 0$), we may simply determine the resistance of the filtration barrier and eliminate the pressure P_1 :

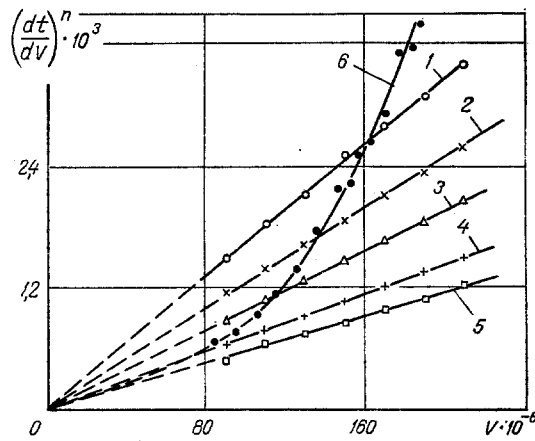


Fig. 2. Filtration of suspensions with various properties of the deposit [6]: carboxymethylcellulose - silicon dioxide, $n = 0.515$, $c = 120$ g/liter; pressure: 1) 5 tech. atm, 2) 7; 3) 9; 4) 12; 5) 15; carboxymethylcellulose - aluminum oxide, $n = 0.467$, $c = 175$ g/liter; 6) 5 tech. atm. $(dt/dV)^n \cdot 10^3$, $(\text{sec}/\text{m}^3)^n$; V , m^3 .

$$P_1 = kR_m u_1^n. \quad (10)$$

Using the relationship $u_1 = dV/dt$ we obtain the differential filtration equation

$$dt = \left(\frac{kc}{P} \right)^{1/n} (\chi_R)^{1/n} (V + V_m)^{1/n} dV, \quad (11)$$

$$V_m = \frac{R_m}{c\chi_R}.$$

For isobaric filtration, after integration over the ranges ($V = 0, V$) and ($t = 0, t$), Eq. (11) takes the form [6]

$$(V + V_m)^{\frac{n+1}{n}} - V_m^{\frac{n+1}{n}} = \left(\frac{1}{\chi_R} \right)^{1/n} \left(\frac{n+1}{n} \right) \left(\frac{p}{kc} \right)^{1/n} t. \quad (12)$$

If filtration at constant pressure is preceded by a transient period, after integrating Eq. (11) over the ranges (V_0, V) and ($0, t$) we obtain

$$(V + V_m)^{\frac{n+1}{n}} - (V_0 + V_m)^{\frac{n+1}{n}} = \left(\frac{1}{\chi_R} \right)^{1/n} \left(\frac{n+1}{n} \right) \left(\frac{p}{kc} \right)^{1/n} t \quad (13)$$

or

$$t = (\chi_R)^{1/n} \left(\frac{n}{n+1} \right) \left(\frac{kc}{p} \right)^{1/n} \left[(V + V_m)^{\frac{n+1}{n}} - V_m^{\frac{n+1}{n}} \right] + \varphi(V_0, V_m). \quad (14)$$

Kozicki et al. [2] used their own model of the motion through the layer, allowing for the slip velocity, and obtained the equation of isobaric filtration in the form

$$t \left(\frac{n}{n+1} \right) \left(\frac{P}{kc} \right)^{1/n} = \kappa \left[(V + V_m)^{\frac{n+1}{n}} - V_m^{\frac{n+1}{n}} \right], \quad (15)$$

where

$$\kappa = \gamma_R^{\frac{1}{n}} \left(1 - \frac{u}{u_p} \right), \quad V_m = \frac{R_m}{c\gamma_R}, \quad (15a)$$

and also

$$\gamma_x = \alpha_x \left[\frac{k_i (1 - \epsilon_x) S}{\epsilon_x^2} \right]^{n-1} \left[\frac{(1 + n\xi)}{n(1 + \xi)} \right]^n. \quad (15b)$$

The possibility of using Eqs. (12) and (15) has been demonstrated experimentally for filtration systems with not too high a concentration of the solid phase in polymer solutions of low concentration [1, 2, 4, 6]. In these systems the empirical characteristics of the resistance of the layer χ_R or κ maintain constant values during the filtration process (Fig. 2, straight lines 1-5).

It is reasonable to assume on the basis of [1, 4, 6] that the absorption equilibrium established in the filtration suspension remains intact during the whole filtration process. In view of this, when describing the properties of the liquid during filtration, it is justifiable to use the rheological parameters of the filtrate.

The filtration of highly concentrated suspensions was considered in [5], using the method of Tiller and Shirato [11, 10] for non-Newtonian liquids. A filtration correction factor J_n was introduced into the solution, allowing for the change in the velocity of the liquid through the compressible deposit. The differential filtration equation then took the simple form

$$u_1^n = \left(\frac{dV}{dt} \right)^n = \frac{(P - P_1)}{k\alpha_R J_n \omega} \quad (16)$$

The average specific resistance α_R is given by Eqs. (7) and (9). As before, introducing the concept of the resistance of the filtration barrier, Eq. (16) may be put in the form

$$\left(\frac{dt}{dV} \right)^n = \frac{1}{\omega} \left(\frac{n+1}{n} \right) (V + V_m), \quad (17)$$

where

$$\omega = \left(\frac{n+1}{n} \right) \left(\frac{P}{k\alpha_R J_n c} \right)^{1/n}$$

In a general consideration of the question it should be emphasized that the structure of the deposit in the case of non-Newtonian filtration differs from that encountered in ordinary liquids. It is reasonable to consider that this is due to the adsorption of the polymer on the surface of the solid particles. Thus, on the basis of our present experiments alone we cannot determine the mechanical properties of the deposit, i.e., its resistance, compressibility, etc., by using measurements of the filtration of suspensions in ordinary liquids.

For certain filtration systems, especially those with a higher concentration of the polymer in the solution, Eqs. (12) and (15) are inapplicable (curve 6, Fig. 2). In view of this, the authors of [5] proposed an isobaric filtration equation based on a model according to which the complete cycle of isobaric filtration consists of the following stages: a short initial period when the pressure P_1 over the filtration barrier is approximately equal to the pressure p (in this period α_R , J_n , n) while the filtration pressure varies with time; the main period of filtration in which P_1 is considerably smaller than p while the parameters α_R , J_n , n , and p are, in principle, constant. In view of this, Eq. (17) allows for the initial period by introducing the arbitrary function $F(V)$:

$$\begin{aligned} F(V) &\neq 0, & 0 < V < V_i, \\ F(V) &= 0, & V > V_i. \end{aligned} \quad (18)$$

After integration over the ranges $0-V$ and $0-t$, by analogy with Eq. (14) we obtain

$$t = \frac{1}{\omega} \left[(V + V_m)^{\frac{n+1}{n}} - V_m^{\frac{n+1}{n}} \right] + t_0. \quad (19)$$

Expanding (19) in series and making some simplifications, the authors in question obtained the relationship

$$t = \frac{1}{\omega} V^{\frac{n+1}{n}} + NV^{\frac{1}{n}} + t_0, \quad (20)$$

where

$$N = \frac{n+1}{n} \cdot \frac{V_m}{\omega} = V_m \left[\frac{k\alpha_R J_n}{Pc} \right]^{1/n}$$

The parameters of Eq. (20), namely, n , ω , N , and t_0 , should be chosen on the basis of the results of filtration measurements. The computing procedure recommended by the authors lies in specifying a set of quantities n and then determining the values of N , ω , and t_0 by the method of least squares. The barrier constant V_m should, in turn, be determined from the results of filtration measurements on an aqueous solution containing the same solid phase. The index n is then a quantity depending on the conditions of the process and cannot be

determined on the basis of independent measurements of the rheological parameters of the liquid.

An analysis of the results of our own investigations showed, however, that the essential cause of the observed phenomena (the course of the process represented by curve 6 of Fig. 2) is the variability of the specific resistance of the deposit during the complete filtration cycle [5, 7, 8].

In order to justify this assertion, it should be emphasized that similar anomalies also appear during the filtration of Newtonian liquids [12-18]. In this case the problematical question as to changes taking place in the index n according to the filtration conditions does not arise. Excluding chemical or biological changes and end effects, we may explain these phenomena by the dependence of the average specific resistance on the duration of the process (or the rate of formation of the deposit). It should be emphasized that when the polymer is adsorbed on the solid particles the deposit formed in non-Newtonian filtration possesses the properties of amorphous greasy slimes.

The model equation of non-Newtonian filtration presented in this paper was accordingly derived from the following premises:

to a first approximation, the index n is constant and corresponds to the value obtained from rheological measurements of the filtrate in the range of slip velocities occurring in the course of filtration;

the specific resistance of the deposit (i.e., its structure) is variable in time over the complete cycle of isobaric filtration and depends on the instantaneous shear velocity (shear stresses) during the formation of the deposit;

the resistance of the filtration barrier, if necessary, is neglected;

the main period of isobaric filtration is preceded by a brief initial period characterized by transient conditions. According to Eq. (11), the differential filtration equation takes the form

$$\left(\frac{dt}{dV}\right)^n = \kappa \left(\frac{kc}{P}\right) (V + V_m). \quad (21)$$

Since according to the assumed model the average specific resistance depends on the slip velocity, it may be represented by the equation

$$\kappa = \kappa_0 + \beta \frac{dt}{dV}. \quad (22)$$

It follows from this equation that a particular value of the specific resistance is taken for the initial instant of filtration. Combining (21) and (22), we obtain

$$\left(\frac{dt}{dV}\right)^n = \left(\frac{kc}{P}\right) \left(\kappa_0 + \beta \frac{dt}{dV}\right) (V + V_m) \quad (23)$$

or

$$\left(\frac{dt}{dV}\right)^n = A \left[1 + B \left(\frac{dt}{dV}\right)\right] (V + V_m). \quad (24)$$

After making the substitution $y = V + V_m$ we obtain the differential equation

$$\left(\frac{dt}{dy}\right)^n = Ay + AB \left(\frac{dt}{dy}\right) y \quad (25)$$

with the initial conditions $t = 0$; $V = 0$; $y = V_m$. The applicability of the equation for describing the filtration of certain suspensions of solid particles in Newtonian liquids was established experimentally in [8].

For non-Newtonian liquids the general equation (25) may be transformed by the substitution

$$\frac{dy}{dt} = \frac{dV}{dt} = h. \quad (26)$$

We thus have

$$1 = Ah^{n-1} (B + h) y. \quad (27)$$

Using the equations

$$\frac{dy}{dh} = \frac{dy}{dt} \cdot \frac{dt}{dh} = h \frac{dt}{dh}, \quad (28)$$

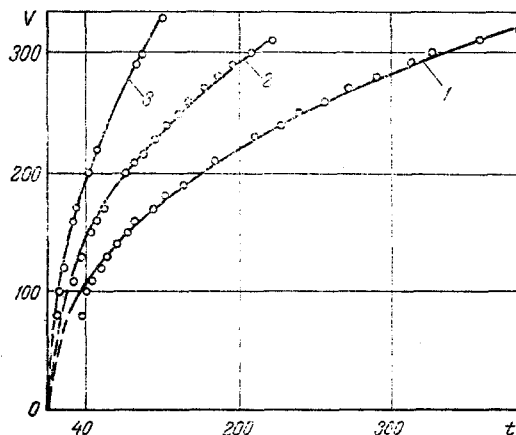


Fig. 3. Filtration of suspensions of calcium carbonate in carboxymethylcellulose ($n = 0.875$). Continuous curve) experimental; points) computed by means of Eq. (36): 1) $m = 33\%$, $p = 3.2$ tech. atm, $A = 6.33 \cdot 10^{-3}$, $B = 0.160$; 2) $m = 33\%$, $p = 5.2$ tech. atm, $A = 3.32 \cdot 10^{-3}$, $B = 0.390$; 3) $m = 33\%$, $p = 9$ tech. atm, $A = 1.86 \cdot 10^{-3}$, $B = 0.537$. t, sec.

we obtain

$$h \frac{dt}{dh} = \frac{d}{dh} \left[\frac{1}{Ah^{n-1}(B+h)} \right]. \quad (29)$$

From Eq. (29) we may obtain an integroparametric solution of the general form of the differential equation (25):

$$t = \int \frac{1}{h} \left\{ \frac{d}{dh} \left[\frac{h^{1-n}}{A(B+h)} \right] \right\} dh, \quad (30)$$

$$y = \frac{h^{1-n}}{A(B+h)}. \quad (31)$$

Substituting

$$g = \frac{1}{h} = \frac{dt}{dy} = \frac{dt}{dV} \quad (32)$$

and remembering the initial conditions

$$t = 0, \quad V = 0, \quad y = V_m,$$

from Eq. (31) we find the value of g_0 such that V_m equals

$$V_m = \frac{g_0^n}{A(1 + Bg_0)}. \quad (33)$$

Allowing for the limits of integration, Eq. (30) thus takes the form

$$t = \int_{g_0}^g \frac{[n - (1-n)Bx] x^n dx}{1(1+Bx)}, \quad (34)$$

where

$$y = V + V_m = \frac{g^n}{A(1+Bg)}.$$

Subject to the condition $V_m = 0$, Eq. (34) may be brought to a form convenient for numerical calculations. Denoting

$$\int_0^g \frac{[n - (1-n)Bx] x^n dx}{A(1+Bx)^2} = -t_0, \quad (35)$$

we assume that the constant represents the initial period of transient isobaric filtration by analogy with (20). Equation (34) may be expressed in the form

$$t = \int_0^g \frac{[n - (1-n)Bx] x^2 dx}{A(1+Bx)^2} + t_0 \quad (36)$$

and regarding the resistance of the fabric as $V_m \approx 0$, we obtain

$$y = V = \frac{g^n}{A(1+Bg)}$$

The constant t_0 represents the initial transient period of isobaric filtration by analogy with Eqs. (14) and (20). The use of Eq. (36) to describe filtration is illustrated by the example of Fig. 3. The continuous curve gives the experimental data (19); the points were computed by means of Eq. (36), with a numerical determination of the parameters A , B , and t_0 . We consider that Eq. (36) may be used as a model equation for describing the non-Newtonian filtration of several types of systems.

NOTATION

A , B , coefficients; c , concentration of suspension in 1 g of solid phase per 1 cm³ of liquid; $e = \epsilon/(1 - \epsilon)$, porosity function; g , h , variables; J_n , filtration factor; K , permeability of deposit; k , consistency coefficient; k_1 , parameter of porous layer; m , weight percent of solid phase in suspension; N , function; n , flow index; p , pressure in liquid; P , pressure of liquid over deposit; P_1 , pressure over filtration barrier; P_0 , pressure under filtration barrier; p_s , pressure compressing the deposit; q , apparent filtration velocity; R_m , resistance of filtration barrier; r , apparent velocity of solid particles in deposit; S , specific surface; T , curvature coefficient; t , time; t_0 , function; u_1 , apparent filtration velocity at the level of the filtration barrier; u_p , apparent slip velocity; V , volume of filtrate per unit surface of the filtration barrier; V_m , constant of filtration barrier; V_0 , volume of filtrate in the initial period of isobaric filtration; w , weight of solid in the deposit per unit filtration surface; x , coordinate, height; y , variable; α , specific resistance of deposit for Newtonian filtration; α_R , average specific resistance of deposit; β , coefficient; $\dot{\gamma}$, shear velocity; γ , generalized resistance[2]; ϵ , porosity; χ , generalized resistance; χ_R , average generalized resistance; μ_{ef} , effective viscosity; ξ , parameter of porous layer; ρ_s , density of solid phase; τ , shear stress; ω , κ , functions.

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